Reg.No. \_\_\_\_\_\_\_\_\_\_\_\_



**UNIVERSITY**

(Karunya Institute of Technology & Sciences)

(Declared as Deemed-to-be University under Sec.3 of the UGC Act, 1956)

**End Semester Examination – Nov/Dec – 2016**

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | **Semester :** | **2016-17 ODD** |
| **Code :** | **15MA3009** | **Duration :** | **3hrs** |
| **Sub. Name :** | **Field Thoery** | **Max. marks :** | **100** |

**ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Q. No.** | **Sub Div.** | **Questions** | **Course**  **Outcome** | **Marks** |
| 1. | a. | Prove that *every finite division ring is a field.* | CO1 | 20 |
| (OR) | | | | |
| 2. | a. | If *F* is a finite field and *α ≠ 0* and *β ≠ 0* are two elements of *F,* then prove that there exist *a* and *b* in *F* such that *1+αa2 + βb2 = 0.* | CO1 | 14 |
| b. | If the finite field *F* has *pm*elements, the prove that every element *a* in *F* satisfies *ap = a.* | CO1 | 6 |
| 3. | a. | If *f(x)* and *g(x)* are two non-zero elements of *F[x],* then prove that  *deg(f(x)g(x)) = deg f(x) + deg g(x).* | CO2 | 10 |
|  | b. | Prove that any polynomial in *F[x]* can be written in the unique manner as a product of irreducible polynomials in *F[x]* | CO2 | 10 |
| (OR) | | | | |
| 4. | a. | State and prove *Eisenstein criteria for polynomial .* | CO2 | 10 |
|  | b. | Prove that the product of two primitive polynomials is again primitive. | CO2 | 10 |
| 5. | a. | If *K* is a finite extension field of *F* and *L* is a finite extension of *K,* then prove that *L* is a finite extension of *F* and *[L:F] = [L:K][K:F]* | CO2 | 20 |
| (OR) | | | | |
| 6. | a. | If an element *a* in *K* is algebraic over *F*, then prove that there exists a unique monic polynomial *P(x)* of positive degree over *F* such that *i. P(a) = 0*  ii. If for any polynomial *f(x)* in *F[x]* with *f(a) = 0,* then *P(x)* divides *f(x).* | CO2 | 20 |
| 7. | a. | Find the deg of the extension of the splitting field of *x3 – 2 ε Q[x]* | CO2 | 10 |
|  | b. | State and prove the exsistence of splitting field for any non-constantpolynomial in *F[x],* where *F* is a field. | CO2 | 10 |
| (OR) | | | | |
| 8. | a. | Prove that *K* is a normal extension of *F* if and only if *K* is the splitting field of some polynomial over *F.* | CO3 | 20 |
|  | | **Compulsory:** |  |  |
| 9. | a. | State and prove the *fundamental theory of Galois group* | CO3 | 20 |

**Course Outcomes:** Students will be able to understand the proof techniques in

CO 1: Wedderburn Theorem on Finite Division Ring,

CO 2 : Eisenstein Irreducible Criterian,

CO 3: Solvability by radicals.

ALL THE BEST